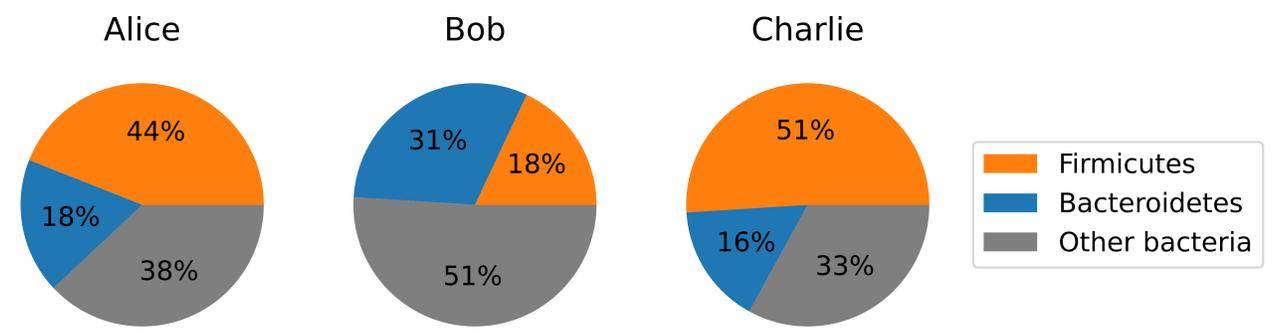


Compositional Data

Compositional Data (CoDa) describe the parts of a whole:

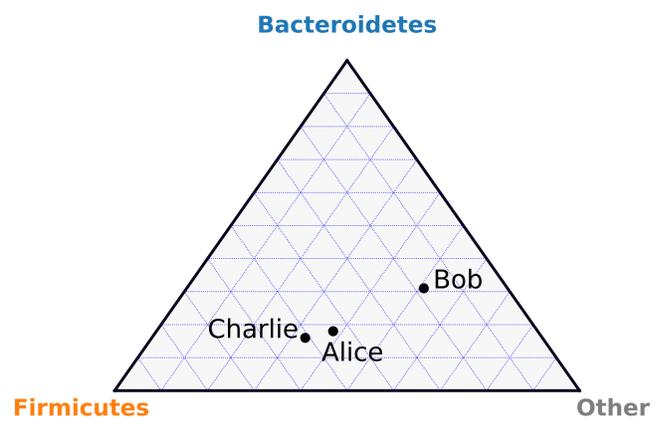


Equivalently, CoDa can be thought of as simplex-valued data:

$$\mathcal{D} = \{\mathbf{x}_i \in \Delta^K\}_{i=1}^n,$$

where Δ^K denotes the simplex:

$$\Delta^K = \left\{ \mathbf{x} \in \mathbb{R}_+^K : \sum_{k=1}^K x_k = 1 \right\}.$$



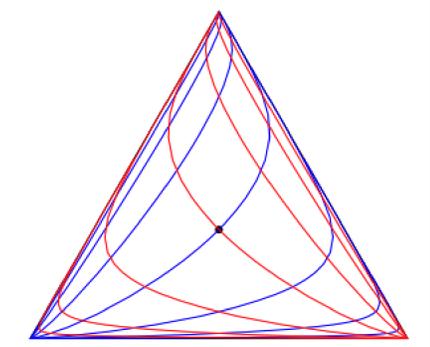
Methods: Aitchison Mixup

Aitchison (1984) established the following Hilbert space structure on the simplex:

$$\mathbf{v} \oplus \mathbf{x} = \frac{1}{\sum_{j=1}^p v_j x_j} [v_1 x_1, \dots, v_p x_p],$$

$$\lambda \odot \mathbf{x} = \frac{1}{\sum_{j=1}^p x_j^\lambda} [x_1^\lambda, \dots, x_p^\lambda],$$

$$\langle \mathbf{v}, \mathbf{x} \rangle = \frac{1}{2p} \sum_{j=1}^p \sum_{k=1}^p \log \left(\frac{v_j}{v_k} \right) \log \left(\frac{x_j}{x_k} \right).$$



Aitchison Geometry on Δ^2
(Orthogonal grid)

Given training points \mathbf{x}_1 and \mathbf{x}_2 , and $\lambda \sim U(0, 1)$, generate:

$$\mathbf{x}^{\text{aug}} := (\lambda \odot \mathbf{x}_1) \oplus ((1 - \lambda) \odot \mathbf{x}_2)$$

Experiments: Supervised Learning

Aitchison Mixup improves existing microbiome learning pipelines across 12 standard benchmarks.

Task	RF	Aug	XGB	Aug	mAML	Aug	DeepCoDa	Aug	MetaNN	Aug
Crohn's (Ileum)	0.72	0.79	0.76	0.79	0.72	0.74	0.73	0.79	0.74	0.74
Crohn's (Rectum)	0.78	0.82	0.81	0.80	0.80	0.80	0.78	0.83	0.74	0.74
Gastrointestinal	1.00									
Female/Male	0.60	0.64	0.57	0.57	0.56	0.58	0.58	0.58	0.50	0.51
Stool/Tongue	1.00									
Plaque	0.81	0.83	0.82	0.83	0.84	0.83	0.78	0.82	0.75	0.76
Colorectal Cancer	0.68	0.67	0.67	0.69	0.73	0.74	0.63	0.73	0.59	0.54
Diabetes	0.62	0.65	0.66	0.68	0.64	0.65	0.45	0.70	0.64	0.64
Cirrhosis	0.93	0.93	0.94	0.95	0.92	0.93	0.84	0.90	0.76	0.82
Black/Hispanic	0.53	0.60	0.57	0.61	0.61	0.62	0.62	0.63	0.63	0.61
Nugent Score	0.98	0.96	0.95							
Black/White	0.55	0.61	0.58	0.65	0.61	0.61	0.66	0.65	0.58	0.60
Mean	0.77	0.79	0.78	0.80	0.78	0.79	0.75	0.80	0.74	0.74

The Human Microbiome



Preterm Birth Prediction: Microbiome
DREAM Challenge

Logos: NIH, Wayne State School of Medicine, Michigan Medicine, University of Colorado Anschutz Medical Campus, SageBionetworks, UCSF, Stanford University, March of Dimes.



Heart Failure Prediction: Microbiome
FINRISK DREAM Challenge

Logos: Finnish Institute for Health and Welfare, University of Turku, Baker, University of Cambridge, Heidelberg Faculty of Medicine, Informatics for Life, Microbiome, UC San Diego.

Also in the paper but not shown here...

- ▶ Compositional Feature Dropout.
- ▶ Compositional CutMix.
- ▶ Contrastive Learning.

Acknowledgements

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